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Electroweak Tests Using Leptonic Decays of the Z ^{0}

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Electroweak tests using leptonic decays of the Z^0

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LEP results characterizing the reactions $e^+e^- \rightarrow Z^0 \rightarrow e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ are presented. Measurements of the partial widths for these reactions, and of their differential cross sections as a function of polar angle permit a determination of electroweak parameters. Comparisons between these measured electroweak parameters and the Standard Model predictions are a precise test of the Standard Model. Alternatively, it can be assumed that the Standard Model is correct, in which case the data can be used to infer a mass for the top quark.

1. Introduction

The Large Electron Positron (LEP) storage ring has recently been completed at CERN, and has been producing Z^0 particles through the annihilation of electrons and positrons. The Z^0 particle lives for about 10^{-25} s before it decays into a fermion–anti-fermion pair. The majority of these decays are into a quark–antiquark pair. However, about 10% of the time the Z^0 decays leptonically, into a $\mu^+\mu^-$ pair, an e^+e^- pair or a $\tau^+\tau^-$ pair. The experimental signature of these leptonic final states is particularly clear. In the case of μ and e one has only two charged tracks and their associated calorimeter hits in the detector, with possibly one or two additional clusters of calorimeter hits. The experimental signature of a Z^0 decay into taus is also quite clean (but less so than for μ or e), with few charged tracks and calorimeter hits registered in the detector.

The production of a Z^0 and its subsequent decay into leptons is also well understood in theory. The relevant theory is the 'Standard Model' of the unified electromagnetic and weak forces. This unified force is called the electroweak force (Glashow 1961; Salam 1968; Weinberg 1967). Careful study of the leptonic decay products of the Z^0 allow us to probe the structure of the electroweak force and test the predictions of the Standard Model.

2. The Standard Model predictions

The basic process for Z^0 production and decay into fermions can be represented by the diagram shown in figure 1. In the case of an e^+e^- final state the situation is complicated by the presence of extra diagrams due to the t-channel contribution. A t-channel contribution arises in processes where the initial state particles are identical to the final state particles; the relevant diagrams for the electron case are shown in figure 2.

There are really two processes represented by the diagram in figure 1, one where the fermions couple to a photon, and the other where they couple to a Z^0. The Standard Model tells us how the particles in these diagrams couple to each other. It

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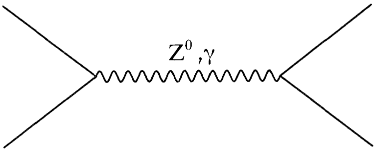

Figure 1. The Born level diagrams for fermion production, the wavy line can be either a photon or a Z^0.

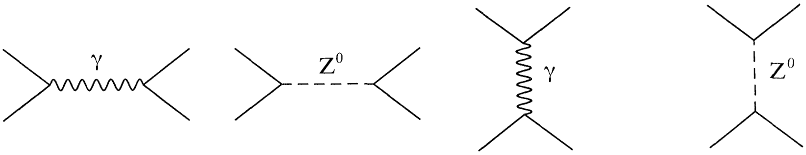

Figure 2. The Born level diagrams for electron production in electron-positron annihilation (the Bhabha process).

tells us that as a function of the centre-of-mass energy of the electron and positron ($\sqrt{s}$), the cross section (σ) for producing a fermion f (not an electron) goes as

$$\sigma = \frac{s(12\pi\Gamma_{\mathrm{ff}}\Gamma_{\mathrm{ee}}/M_Z^2 + I_{\mathrm{f}}N_{\mathrm{c}}(s-M_Z^2)/s)}{(s-M_Z^2)+s^2\Gamma_Z/M_Z^2} + \frac{4\pi N_{\mathrm{c}}Q_{\mathrm{f}}^2\alpha^2(s)}{3s}, \tag{1}$$

where Γ_{ff} is the partial width for decays into leptons of type f, N_{c} is a colour factor, Q_{f} is the electric charge of fermions of type f, α is the fine structure constant, and I_{f} gives the magnitude of the interference. Note that the resonance terms are the Z^0 exchange term and the term that gives the strength of the interference between the Z^0 and the γ exchange contributions, the first two terms in (1) respectively. This expression also holds for the electron channel when one considers only the s-channel contribution. To get the full cross-section for Bhabha scattering one must add additional terms to (1) to take into account the t-channel contribution and its interference with the s-channel.

The quantities M_Z, Γ_Z that appear in (1) are not the true, physically observable values of the mass and width of the Z^0, they are known as the 'bare' values and are the values that appear in the lagrangian formulation of the Standard Model.

To get useful predictions from the Standard Model we have to include two classes of higher order diagrams that provide radiative corrections to the first order 'tree level' diagram shown in figure 1 (see, for example, Altarelli *et al.* 1989). The largest corrections are the QED, or photonic corrections, that come from diagrams of the form shown in figure 3. These corrections are as large as 30% on the resonance, and are important corrections in view of the fact that the experimental error on the peak cross section is $O(1\%)$. The second class of corrections are the electroweak, or non-photonic, corrections that come from the kind of diagrams shown in figure 4. These corrections are also large but they are smaller than the photonic corrections.

If one calculates the contribution to the process in figure 1 from any single diagram of the type shown in figure 3 or figure 4, one finds that it is infinite! However, if one calculates all diagrams, up to a given order, and sums the results one finds a finite value for the cross section, $\sigma(e^+e^- \rightarrow \gamma Z^0 \rightarrow f\bar{f})$. It turns out that negative infinities in some of the diagrams cancel with positive infinities in other diagrams.

This magic is called renormalization. Remarkably, after we have applied this magic, if we keep only the dominant (leading log) corrections, we may write the cross

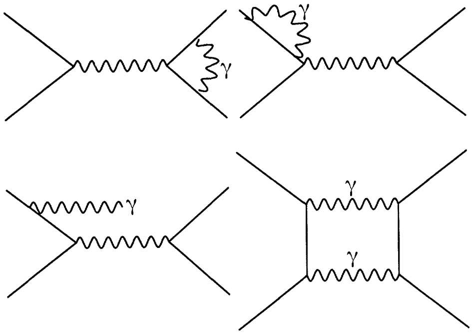

Figure 3. Photonic diagrams contributing to the radiative corrections of the Born diagram.

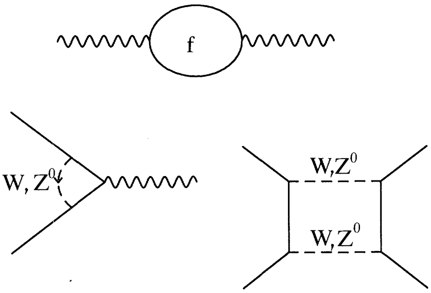

Figure 4. Diagrams contributing to the non-photonic radiative corrections to the Born diagram.

section in exactly the same form as (1) simply by replacing the quantities that appear in it with their renormalized counterparts. These new quantities are the physically observable values of the Z^0 mass, width, etc. The renormalization procedure absorbs the radiative corrections into 'running coupling constants' and renormalized masses which are the appropriate physically observable couplings and masses of the theory.

One renormalized version of (1) is called the modified born expression, or MBE. The Standard Model predicts various relationships between the quantities that appear in the MBE, i.e. the renormalized couplings and physical masses. In fact, next to leading log terms are not included in the MBE, but are large enough that they have to be taken into account. These terms typically have to be evaluated by Monte Carlo methods, and several different Monte Carlo packages have been used for this purpose, the most commonly used being the ZFITTER package written by the Zeuthen–Dubna collaboration (Bardin *et al.* 1989, 1991*a*,*b*; Bilenky 1989).

One of the aims of the LEP physics programme is to accurately measure the physical quantities that appear in these relations, and thus test the Standard Model. This cannot be done with LEP data alone, since the relations contain masses like M_W (the W boson mass), M_t (the mass of the yet to be discovered top quark) and M_H (the mass of the, also not yet seen, Higgs particle). This limits our ability to test the Standard Model. What we can do, however, is turn the problem around; measure the quantities that appear in the Standard Model and then make predictions for M_t, M_H, etc. If these predicted masses are excluded by experiment, the model will have failed. If on the other hand the top or Higgs are found with the predicted mass we will take this as a triumph for the theory.

Table 1. *The number of Z decays into the various lepton channels seen by the* LEP *detectors*

channel	ALEPH	DELPHI	L3	OPAL	tot. LEP
e^+e^-	9717	2615	4175	5415	21922
$\mu^+\mu^-$	5930	2489	3245	7240	18904
$\tau^+\tau^-$	5581	2039	2540	5559	15719

Table 2. *Results from* LEP *for M_Z, Γ_Z, σ(peak) and Γ_{had} together with their Standard Model predictions*

(An error of 0.020 GeV has been added to the average Z^0 mass to take into account the uncertainty in the LEP machine energy.)

	M_Z/GeV	Γ_Z/GeV	σ(peak)/nb	Γ_{had}/MeV
ALEPH	91.182 ± 0.009	2.488 ± 0.017	41.76 ± 0.39	1756 ± 15
DELPHI	91.175 ± 0.010	2.454 ± 0.020	41.98 ± 0.63	1718 ± 22
L3	91.180 ± 0.010	2.500 ± 0.017	40.93 ± 0.45	1745 ± 23
OPAL	91.160 ± 0.009	2.497 ± 0.017	41.23 ± 0.47	1747 ± 19
LEP ave.	91.174 ± 0.021	2.487 ± 0.009	41.44 ± 0.28	1744 ± 10
S.M.	—	2.487 ± 0.022	41.48 ± 0.10	1726 ± 8

3. The results from LEP

The number of Z^0 decays to muons, electrons and taus for all four of the LEP experiments to the end of 1990 are summarized in table 1. Over 56000 leptonic Z^0 decays have been seen at LEP. In this section preliminary results obtained by the four LEP experiments are presented. In the case of L3 and DELPHI these results have been updated since the Aspen conference (ALEPH 1990*a*; DELPHI 1991; L3 1991; OPAL 1990*a*). For OPAL and ALEPH the results presented here are the same as those presented at Aspen. For details about the individual experiment's analyses the reader is referred to the literature (ALEPH 1990; DELPHI 1990; L3 1990; OPAL 1990).

Results from LEP are summarized in the following tables. In table 2 the results for the Z^0 mass and width, the cross section on the peak, and the hadronic width are presented. In these tables the Standard Model predictions are given below the LEP average values. These Standard Model predictions are valid for $M_Z = 91.175 \pm 0.021$ GeV, $\alpha_s = 0.115 \pm 0.009$, a top quark with mass between 100 and 200 GeV and a Higgs of mass between 50 and 1000 GeV. The results given here are obtained by simultaneous fits to the lineshape in the channels $Z^0 \rightarrow$ hadrons, e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$, and assume lepton universality. In evaluating the errors on the LEP average value, the errors on the results from the individual experiments have been added in quadrature, after subtracting (where it enters) an appropriate contribution due to the theoretical error on the luminosity for each experiment. For all the experiments the total systematic error on the luminosity is of order 1%, about half of which is due to theoretical uncertainties. In addition the LEP energy uncertainty 0.020 GeV has been added to the error on M_Z (Hatton *et al.* 1990). Unless otherwise indicated, all errors quoted in these tables are combined systematic and statistical errors.

By dropping the assumption of lepton universality, the individual lepton widths can be fitted, again by fitting to the cross section for hadrons, muons, electrons and taus simultaneously. In this six-parameter fit the hadron data essentially fixes the value of M_Z, and the lepton data fixes the leptonic widths. Results for the leptonic

Table 3. *Results from* LEP *for the leptonic partial widths of the* Z^0

(The error on the average takes into account the theoretical error on the luminosity which is common to each experiment.)

	Γ_e/MeV	Γ_μ/MeV	Γ_τ/MeV	Γ_l/MeV
ALEPH	84.2 ± 0.9	80.9 ± 1.4	82.9 ± 1.6	83.3 ± 0.7
DELPHI	81.6 ± 1.3	88.4 ± 2.4	84.9 ± 2.7	83.4 ± 1.0
L3	83.0 ± 1.0	84.3 ± 2.0	83.3 ± 2.6	83.3 ± 0.8
OPAL	83.4 ± 1.0	83.5 ± 1.5	83.1 ± 1.9	83.4 ± 0.7
LEP ave.	83.3 ± 0.5	83.3 ± 0.9	83.3 ± 1.0	83.3 ± 0.4
S.M.	83.7 ± 0.6	83.7 ± 0.6	83.5 ± 0.6	83.7 ± 0.6

Table 4. *Results from fitting simultaneously to* LEP *cross section and lepton asymmetry data for* g_v^2, g_a^2 *and* $\sin^2\theta_W$

	g_v^2	g_a^2	$\sin^2\theta_W$
ALEPH	0.0020 ± 0.0007	0.248 ± 0.002	0.2325 ± 0.0025
DELPHI	0.0007 ± 0.0014	0.251 ± 0.003	0.2318 ± 0.0033
L3	0.0020 ± 0.0013	0.249 ± 0.003	0.2320 ± 0.0030
OPAL	0.0006 ± 0.0007	0.251 ± 0.002	0.2325 ± 0.0022
LEP ave.	0.0013 ± 0.0004	0.250 ± 0.001	0.2323 ± 0.0014
S.M.	0.0012 ± 0.0004	0.251 ± 0.002	0.2323 ± 0.0033

partial widths of the Z^0 are given in table 3 along with the Standard Model values. The partial widths contain a dependence on the square root of the luminosity, and this has been taken into account when subtracting the common theoretical error on the luminosity during the error calculation.

By assuming lepton universality and fitting to the cross-section data together with the lepton asymmetries, values for the coupling constants g_a and g_v can be obtained. These are presented in table 4, together with values of $\sin^2\theta_W$ obtained from the leptonic widths and the Standard Model predictions for these parameters. No attempt has been made to subtract off the common theoretical uncertainty in the luminosity error for each experiment, which doesn't enter into the calculation of g_a and g_v, but which enters in a non-trivial way into the $\sin^2\theta_W$ calculation.

The agreement between the measured values of the electroweak parameters presented in these tables and the values predicted by the Standard Model is striking.

4. A simultaneous fit to all LEP data

An alternative to the averaging procedure followed above is to combine the cross section and lepton asymmetry data from all the experiments and then refit the lineshape. This procedure takes care of correlations between the various data points and permits a better treatment of common systematic errors. In this section results obtained by this method from a simultaneous fit to the presently available data from all four experiments are presented. The fitting has been performed with the 'ZFITTER' program of Bardin *et al.*

For L3 and DELPHI the data used include the full 1990 running period (L3 1991; DELPHI 1991). For OPAL the full 1990 data are used in the muon and tau channels, however, due to the way that OPAL analyse their electron data it has not been

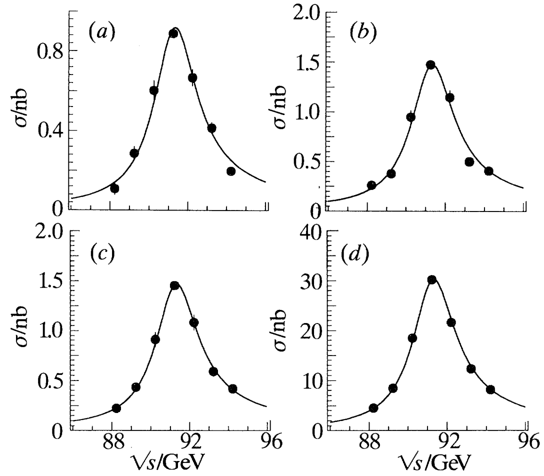

Figure 5. The L3 data together with the result of a combined lineshape fit to the combined LEP cross-section data. (*a*) e^+e^-; (*b*) $\mu^+\mu^-$; (*c*) $\tau^+\tau^-$; (*d*) hadrons.

Table 5. *The results of four- and six-parameter fits to all the available* LEP *cross-section data*

parameter	lept. univ.	no lept. univ.	units
M_Z	91.175 ± 0.005^a	91.175 ± 0.005^a	GeV
Γ_Z	2.491 ± 0.009	2.490 ± 0.010	GeV
Γ_{had}	1.738 ± 0.010	1.737 ± 0.015	GeV
Γ_{lept}	83.7 ± 0.4	—	MeV
Γ_{ee}	—	83.7 ± 0.7	MeV
$\Gamma_{\mu\mu}$	—	83.9 ± 1.0	MeV
$\Gamma_{\tau\tau}$	—	83.1 ± 1.2	MeV

[a] An additional error of 0.020 GeV should be added to take care of the uncertainty in the LEP machine energy.

possible to include the OPAL electron data in the fits (OPAL 1990*a*). This is because OPAL take into account the *t*-channel contribution to the electron channel at the time of fitting and as a result they have not published *t*-channel subtracted cross sections for the electron channel. At the time of writing ALEPH had not yet finalized their results including all the 1990 data and so their most recent published data have been used (ALEPH 1990).

The results of a four-parameter fit to all the cross section data are given in column 1 of table 5. The fit has a χ^2 of 148 for 168 degrees of freedom. From the fitted values of Γ_Z, Γ_e and Γ_{had} we can derive the invisible width and hence the number of light neutrinos. The result is

$$N_\nu = 3.01 \pm 0.05.$$

An alternative is to perform the same fit as above, but with the top quark mass as a free parameter. When this is done, assuming $M_H = 300$ GeV and $\alpha_s = 0.115$ one finds:

$$M_{\text{top}} = 163^{+35}_{-44}\,\text{GeV}.$$

If the assumption of lepton universality is dropped, the fit involves six parameters and returns the individual leptonic widths. The results of this fit are given in the second column of table 5. This fit returns a χ^2 of 147 for 166 degrees of freedom.

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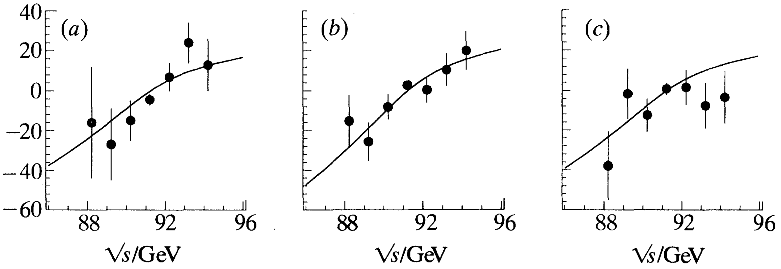

Figure 6. The DELPHI lepton asymmetry data together with the results of a fit to all the available LEP cross-section and lepton asymmetry data. (*a*) e^+e^-; (*b*) $\mu^+\mu^-$; (*c*) $\tau^+\tau^-$.

Table 6. *The results of a combined fit to all the* LEP *cross-section data and all the* LEP *lepton asymmetry data*

parameter	value	units
M_Z	91.175 ± 0.005	GeV
Γ_Z	2.490 ± 0.009	GeV
Γ_{had}	1.738 ± 0.010	GeV
g_a	-0.5031 ± 0.0014	none
g_v	$-0.034^{+0.009}_{-0.007}$	none
$\overline{\sin^2\theta_W}$	0.233 ± 0.004[a]	none
Γ_{lept}	83.5 ± 0.4[a]	MeV
ρ_{eff}	1.0025 ± 0.0028[a]	none

[a] These quantities are derived and are model independent.

The six-parameter fit result together with the L3 cross section data are shown in figure 5.

(*a*) *A combined fit including the lepton asymmetry data*

A fit to all the LEP data for cross sections and the lepton asymmetries returns values for g_a and g_v. The results of such a fit are given in table 6. This fit has a χ^2 of 239 for 272 degrees of freedom. The fit result together with the DELPHI lepton asymmetry data are shown in figure 6.

It is reassuring to see that the results of the combined fits are similar to the results obtained from the simpler averaging procedure used in §3.

5. A combined LEP limit on the top quark mass

For a given Higgs mass, we may assume the Standard Model and lepton universality, and fit for the top quark and Z^0 masses. Three fits have been performed. In the first fit the value of $\sin^2\theta_W$ was left unconstrained, so that it is effectively determined by the LEP data alone. In the second fit $\sin^2\theta_W$ was constrained to be 0.2274 ± 0.0059, which is the average value inferred from W-boson mass measurements made by UA2 and CDF (CDF 1990; UA2 1990; Di Lella 1990). In the third fit $\sin^2\theta_W$ has been constrained to a value of 0.2292 ± 0.0042 which is the average of the collider and neutrino scattering values (CDF 1990; CDHS 1990; CHARM 1987; UA2 1990). The results of these three fits are given in table 7 for a range of Higgs masses between 50 and 1000 GeV.

We may then give the LEP combined limit on the top quark mass as follows, from the LEP data alone:

$$M_{\text{top}} = 163^{+35+15\,M_H=1000}_{-44-26\,M_H=50}\ \text{GeV},$$

Table 7. *The fitted masses, in* GeV, *of the top quark and the* Z, *for various assumed masses of the Higgs, from the combined* LEP *cross-section and asymmetry data*

(The fits assume lepton universality and the Standard Model. In performing the fits α_s was fixed to a value of 0.115 ± 0.009.)

M_H	50 GeV	100 GeV	300 GeV	1000 GeV
$\sin^2\theta_W$ free				
M_Z	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005
M_{top}	137^{+32}_{-38}	147^{+38}_{-50}	163^{+35}_{-44}	178^{+33}_{-40}
$\chi^2/272$	239	239	233	233
$\sin^2\theta_W = 0.2274 \pm 0.0059$				
M_Z	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005
M_{top}	137^{+32}_{-38}	142^{+32}_{-37}	155^{+29}_{-35}	172^{+27}_{-32}
$\chi^2/273$	239	239	239	239
$\sin^2\theta_W$ 0.2292 ± 0.0042				
M_Z	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005	91.175 ± 0.005
M_{top}	123^{+29}_{-33}	129^{+28}_{-33}	143^{+27}_{-32}	161^{+25}_{-30}
$\chi^2/273$	239	239	239	239

from the LEP data together with the UA2 and CDF values of $\sin^2\theta_W$:

$$M_{top} = 155^{+29+17 M_H=1000}_{-35-18 M_H=50}\ \mathrm{GeV},$$

and from the LEP data together with the world average value of: $\sin^2\theta_W$

$$M_{top} = 139^{+27+22 M_H=1000}_{-32-16 M_H=50}\ \mathrm{GeV}.$$

6. Conclusions

All four LEP experiments have measured the Z^0 lineshape parameters and lepton asymmetries with data taken in 1989 and 1990. The results from the four experiments are consistent with each other, and in striking agreement with the predictions of the Standard Model.

Within the framework of the Standard Model, and together with the value of $\sin^2\theta_W$ from the colliders and neutrino–nucleon scattering experiments, the LEP data can be used to infer a limit on the mass of the top quark; $M_t < 263$ GeV at the 95% confidence limit.

It is expected that with the 1991 data from LEP the experiments will reach a point at which the errors on the Standard Model parameters that they measure are limited by systematic errors, rather than statistical errors (this can be expected when each experiment has *ca.* 2×10^6 Z^0s). Further precision tests of the Standard Model will, however still be possible from measurements of tau-polarization and b-quark asymmetries.

I thank all the physicists from all the LEP collaborations who helped me prepare this work, in particular I thank Sunanda Banerjee for actually performing the fits in §5. I also thank the LEP collaborations for making their latest preliminary results available to me ahead of publication. I thank the Royal Society for the invitation to, and hospitality at, their meeting at which this work was presented.

References

ALEPH Collaboration 1990 Measurement of electroweak parameters from Z decays into fermion pairs. *Z. Phys.* C **48**, 365–391.

ALEPH Collaboration 1990*a* ALEPH Z^0 line shape, 1989 and 1990 results. *ALEPH-Note: Electroweak Physics Group, Aspen conference, January 1990.*

Altarelli, G., *et al.* 1989 Z physics at LEP-I. CERN Report *CERN 89-08* (ed. G. Altarelli *et al.*).

Bardin, D., *et al.* 1989 A realistic approach to the standard Z peak. *Z. Phys.* C **44**, 493–502.

Bardin, D., *et al.* 1981*a* Analytic approach to the complete QED corrections to fermion pair production in e^+e^- annihilation. Zeuthen preprint PHE-89-19. *Nucl. Phys.* B. (In the press.)

Bardin, D., *et al.* 1991*b* QED corrections with partial angular integration to fermion pair production in e^+e^- annihilation. Zeuthen preprint PHE-90-17. *Phys. Lett.* B. (In the press.)

Bilenky, M. & Sazonov, A. 1989 The forward backward asymmetry A_{FB} in e^+e^- annihilation. JNIR Dubna preprint *E2-89-792*.

CDF Collaboration 1990 Measurement of the W-boson mass. *Phys. Rev. Lett.* **65**, 2243–2246.

CDHS Collaboration 1990 Electroweak parameters from a high statistics neutrino nucleon scattering experiment. *Z. Phys.* C **45**, 361–379.

CHARM Collaboration 1987 A precise determination of the electroweak mixing angle from semileptonic neutrino scattering. *Z. Phys.* C **36**, 611–628.

DELPHI Collaboration 1990 A study of the leptonic decays of the Z^0-boson. *Phys. Lett.* B **241**, 425–434.

DELPHI Collaboration 1991 DELPHI results on the Z resonance parameters through its hadronic and leptonic decay modes. DELPHI note *DEL-PHI 90-02 PHYS 80, Aspen Conference, January 1991.*

Di Lella, L. 1990 Hadron colliders: past, present and future. CERN preprint *CERN-PPE/90-160*.

Glashow, S. L. 1961 Partial symmetries of weak interactions. *Nucl. Phys.* B **22**, 579–588.

Hatton, V., *et al.* 1990 LEP absolute energy in 1990. CERN, LEP performance note 12.

L3 Collaboration 1990 A determination of electroweak parameters from Z^0 decays into charged leptons. *Phys. Lett.* B **252**, 183–192.

L3 Collaboration 1991 Measurement of electroweak parameters from hadronic and leptonic decays of the Z^0. *L3 Preprint 28.*

OPAL Collaboration 1990 Analysis of Z^0 couplings to charged leptons. *Phys. Lett.* B **247**, 458–472.

OPAL Collaboration 1990*a* Analysis of Z^0 couplings to hadrons and leptons. *OPAL Physics Note 90-21, Aspen Conference, January 1991.*

Salam, A. 1968 Weak and electromagnetic interactions. In *Elementary particle theory* (ed. N. Svartholn), pp. 367–377. Stockholm: Almquist and Wiskell.

UA2 Collaboration 1990 A precise determination of the W and Z masses at the CERN $\bar{p}p$ collider. *Phys. Lett.* B **241**, 150–164.

Weinberg, S. 1967 A model of leptons. *Phys. Rev. Lett.* **19**, 1264–1266.